

# Contents

<b>List of Figures</b>	<b>v</b>
<b>List of Tables</b>	<b>vii</b>
<b>List of Algorithms</b>	<b>ix</b>
<b>List of Acronyms</b>	<b>xi</b>
<b>List of Symbols</b>	<b>xiii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Motivation . . . . .	1
1.2 Structure of this Thesis . . . . .	5
1.3 System Setup . . . . .	7
<b>2 Mathematical Preliminaries</b>	<b>9</b>
2.1 Matrix Theoretic Preliminaries . . . . .	9
2.1.1 General Matrix Pencils . . . . .	9
2.1.2 Even Matrix Pencils and their Condensed Forms . . . . .	14
2.1.3 Skew-Hamiltonian/Hamiltonian Matrix Pencils . . . . .	19
2.2 System Theoretic Basics . . . . .	22
2.2.1 Descriptor Systems, Behaviors, and Stability . . . . .	22
2.2.2 Controllability, Stabilizability, Observability, and Detectability	24
2.2.3 Frequency Domain Analysis . . . . .	28
Laplace Transformation and Transfer Functions . . . . .	28
Polynomial and Rational Matrices . . . . .	30
Some Rational Function Spaces . . . . .	33
2.2.4 Zero Dynamics and Outer Systems . . . . .	34
<b>3 Linear-Quadratic Control Theory for Differential-Algebraic Equations</b>	<b>38</b>
3.1 Introduction . . . . .	38
3.2 State and Feedback Transformations . . . . .	45
3.3 Kalman-Yakubovich-Popov Lemma . . . . .	48
3.4 Even Matrix Pencils and Descriptor Lur'e Equations . . . . .	53
3.5 Stabilizing, Anti-Stabilizing, and Extremal Solutions . . . . .	70

3.6	Spectral Factorization . . . . .	72
3.7	Nonpositive Solutions . . . . .	74
3.8	Linear-Quadratic Optimal Control . . . . .	84
3.8.1	Optimal Control with Zero Terminal Condition . . . . .	84
3.8.2	Optimal Control with Free Terminal Condition . . . . .	106
3.9	Dissipative and Cyclo-Dissipative Systems . . . . .	109
3.9.1	Systems with General Quadratic Supply Rates . . . . .	109
3.9.2	Contractive and Passive Systems . . . . .	120
3.10	Normalized Coprime Factorizations . . . . .	124
3.11	Inner-Outer Factorizations . . . . .	134
3.12	Summary and Outlook . . . . .	141
<b>4</b>	<b>Systems with Counterclockwise Input/Output Dynamics</b>	<b>144</b>
4.1	Introduction . . . . .	144
4.2	Systems with Counterclockwise Input/Output Dynamics and Negative Imaginary Transfer Functions . . . . .	145
4.3	Spectral Characterizations for Negative Imaginariness . . . . .	148
4.4	Enforcement of Negative Imaginariness . . . . .	152
4.4.1	Some Useful Results . . . . .	152
4.4.2	Choice of the New Frequencies . . . . .	155
4.4.3	Choice of the System Norm . . . . .	156
4.4.4	Enforcement Procedure . . . . .	159
4.4.5	Enforcing Symmetry of the Polynomial Part . . . . .	161
4.4.6	The Overall Process . . . . .	161
4.4.7	Reformulation of the Odd Eigenvalue Problem . . . . .	162
4.4.8	Computation of the Eigenvectors . . . . .	164
	The Algorithm . . . . .	164
	Implementation and Numerical Experiments . . . . .	168
4.4.9	An Illustrative Example . . . . .	171
4.5	Conclusions and Outlook . . . . .	173
<b>5</b>	<b>Computation of the Complex Cyclo-Dissipativity Radius</b>	<b>175</b>
5.1	Introduction . . . . .	175
5.2	The Cyclo-Dissipativity Radius . . . . .	177
5.3	Perturbations of the Singular Part and the Defective Infinite Eigenvalues	180
5.3.1	Perturbation of the Singular Part . . . . .	180
5.3.2	Perturbation of the Defective Infinite Eigenvalues . . . . .	186
5.4	Computation of the Complex Cyclo-Dissipativity Radius . . . . .	193
5.5	Numerical Results . . . . .	199
5.5.1	Illustrative Examples . . . . .	199
5.5.2	Limitations of the Method . . . . .	202

5.6	Summary and Open Problems . . . . .	204
<b>6</b>	<b>Computation of the <math>\mathcal{H}_\infty</math>-Norm for Large-Scale Descriptor Systems</b>	<b>205</b>
6.1	Introduction . . . . .	205
6.2	The Pseudopole Set Approach . . . . .	208
6.2.1	Complex $\mathcal{H}_\infty$ -Radius and $\mathcal{H}_\infty$ -Norm . . . . .	208
6.2.2	Computation of the $\varepsilon$ -Pseudopole Set Abscissa . . . . .	214
	Derivation of the Basic Algorithm . . . . .	214
	Choice of the Eigenvalues . . . . .	218
	Algorithmic Details . . . . .	220
	Fixed Point Analysis . . . . .	221
	Local Convergence and Error Analysis . . . . .	224
6.2.3	Newton's Method for Computing the Complex $\mathcal{H}_\infty$ -Radius . . . . .	225
6.2.4	Comparison with the Method of Guglielmi, Gürbüzbalaban, and Overton . . . . .	226
6.2.5	Numerical Results . . . . .	229
	Test Setup . . . . .	229
	Test Results . . . . .	229
	Comparison of Dominance Measures . . . . .	231
	Comparison with Other Methods . . . . .	233
	Limitations of the Method . . . . .	235
6.3	The Even Pencil Approach . . . . .	238
6.3.1	Theoretical Preliminaries and Algorithm Outline . . . . .	238
6.3.2	Structured Iterative Eigensolvers . . . . .	239
6.3.3	Implementation Details . . . . .	241
6.3.4	Numerical Results . . . . .	243
	Test Setup . . . . .	243
	Test Results and Comparison with the Pseudopole Set Approach . . . . .	243
6.4	Conclusions and Future Research Perspectives . . . . .	245
<b>7</b>	<b>Summary and Outlook</b>	<b>248</b>
	<b>Bibliography</b>	<b>251</b>
	<b>Index</b>	<b>269</b>
	<b>Theses</b>	<b>275</b>
	<b>Statement of Scientific Cooperations</b>	<b>279</b>
	<b>Declaration of Honor/Schriftliche Ehrenerklärung</b>	<b>283</b>