Abstract

The present thesis concerns the cooperative control of networked vehicles. Autonomous driving is a topic that is currently being discussed with great interest from researches, vehicle manufactures and the corresponding media. Future autonomous vehicles should bring the passengers to their desired destination while improving both safety and efficiency compared to current human-driven vehicles. The inherent problem of all vehicle coordination tasks is to guarantee collision avoidance in every situation. To this end, autonomous vehicles have to share information with each other in order to perform traffic manoeuvres that require the cooperation of multiple vehicles.

By providing analysis tools and design methods, the theory of networked control systems adds to the development of vehicle controllers with which a collaboration of multiple networked vehicles shows the desired cooperative behaviour. The general idea, which is pursued in this thesis, is to impose requirements (e. g. collision avoidance) on the overall behaviour of the controlled vehicles. Based on these requirements, properties of the controlled vehicles on the one hand and properties of the communication network on the other hand are derived. The local controller of each vehicle and the structure of the communication network have to be designed to meet specific design objectives so that the combination of the controlled vehicles and the communication network achieves the desired overall behaviour.

This procedure is applied to the control of networked vehicles in different ways. At first, scenarios that can be encountered in real world traffic are considered. As a fundamental problem, vehicle platooning is studied extensively which describes the task of arranging a set of vehicles so that they drive with a common velocity and a prescribed distance. Using the procedure described above, local design objectives are derived that have to be satisfied by the vehicle controllers. In particular, it is shown that the vehicles have to be externally positive to achieve collision avoidance. However, the problem of rendering a feedback loop externally positive has been unsolved so far. The present thesis solves the design problem for a general class of linear vehicle models based on a state feedback. The proposed controller structure and parametrisation is shown to achieve the desired behaviour with information that can be gathered via local sensors or the communication network.

As an abstraction from real traffic scenarios, swarms of mobile systems are considered. The main difference between swarming and traffic problems is the freedom of movement since a swarm of mobile systems is not bound to move on prescribed roads. An important implication of this circumstance is that a communication structure that has been appropriate in the beginning might become unsuited for the control task due to the relative movement of the mobile systems. To solve this problem, this thesis proposes to use the Delaunay triangulation as a switching communication structure of a networked system. To this end, it will be shown that the Delaunay triangulation can be maintained by the mobile systems with distributed algorithms.

The task to be solved by the swarm of mobile agents is to achieve a distance-based formation. Due to the free motion of the mobile systems on a driving surface, nonlinear local controllers are applied that perform a gradient descent along an artificial potential field that is composed of relative potential functions. It will be shown how to choose and parametrise the relative potential functions so that the combination of the Delaunay triangulation network structure and the local controllers solve the control task while guaranteeing collision avoidance.

All proposed methods are evaluated with a laboratory plant which consists of several mobile robots that are tracked by a camera system. The experiments verify the theoretical results under consideration of the following aspects. The robots have properties and limitations (e. g. control input bounds, dead band for small inputs, wheel slippage, etc.) that are not considered in the models that describe the nonholonomic kinematics. Nevertheless, the proposed control techniques are shown to be real-time capable and to achieve collision avoidance in all considered scenarios.

1

Introduction to cooperative control of networked vehicles

1.1 Networked control systems

The networking of controlled subsystems will play an increasingly important role in technical automation in the future. Instead of a central control system, each subsystem will have a local controller that is connected to other controllers via a communication network. The exchange of information ensures that a common task can be solved cooperatively. The distributed structure also allows for easy scalability as subsystems are added to or removed from the network.

The present thesis concerns the control of many vehicles that should cooperate with each other to achieve common or individual goals while guaranteeing collision avoidance. From a control-theoretic point of view, the collaboration of multiple subsystems, which represent the controlled vehicles, is represented as a *multi-agent system*. The structure of such a system is presented in Fig. 1.1.

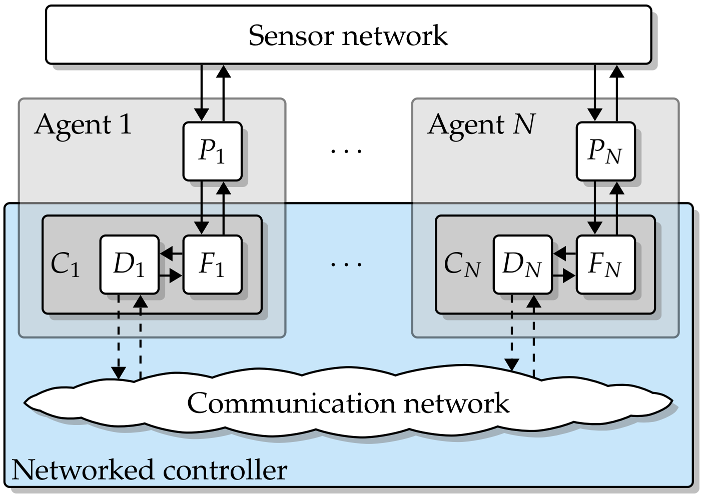

Fig. 1.1: Networked control structure of a multi-agent system

The controlled subsystems are referred to as the *agents* $i = 1, 2, \ldots, N$ and are composed of multiple units. The plant P_i represents the physical behaviour of a vehicle. In order to achieve a desired behaviour of the agents, the local controller C_i determines the input of the plant. To this end, the controller is itself composed of the feedback unit F_i that executes an appropriate control algorithm and the communication and decision unit D_i that is connected to the communication network and collects all information that is needed by F_i to operate.

Due to the connection and interaction of the local controllers, a multi-agent system is said to have a *networked control structure*. The communication and decision units D_i send and receive information via the communication network. The information is not limited to physical quantities as state variables but might also contain intentions of an agent and instructions to other agents. Thus, for the sake of safety and efficiency, the agents are able to make decisions based on their available information and cooperate with other agents if that is required to achieve the individual or common control aim.

In addition to the digital communication network, the agents are coupled via a sensor network which represents the measurement of physical quantities that concern other agents. In the context of networked vehicles, the sensor network represents the coupling of the vehicles via the inter-vehicle distance as an example. The inclusion of physical interactions in the model structure allows for introducing the notion of *cognition* which describes the ability of the agents to sense their environment or other agents that are not connected to the communication network. In contrast to classical physical interconnections as in power plants or processing systems that are coupled statically in close proximity, the sensor network of mobile agents is flexible just like the communication network. That is, the coupling structure can change dynamically.

1.2 Goal of this thesis

The present thesis considers multiple vehicles that should cooperate with each other in order to solve different coordination problems. The fields of application in this thesis to be presented in the next section range from real traffic scenarios to abstract formation problems. In the considered traffic scenarios, the vehicles have individual aims (e. g. to reach a destination) or common goals (e. g. to form a platoon or a formation). The fundamental goal in each situation is to guarantee *collision avoidance*.

> The goal of this thesis is to develop methods for the cooperative control of networked vehicles that achieve collision avoidance and to test them in different traffic scenarios.

This task is considered under the circumstance that there is no global coordinator. Consequently, the vehicles have to cooperate with each other by sharing information via the communication network.

Way of solution

The idea of the design of networked control systems in this thesis is illustrated in Fig. 1.2. First, it is considered which behaviour the overall system should have to achieve an overarching control aim. The result should be a list of requirements on the networked control system. Based on these requirements, it is then elaborated which properties the agents and the communication network should have. The challenge is to find local properties that can be implemented by the local feedback F_i on the one hand and a network structure that can be determined and maintained locally by the communication and decision unit D_i on the other hand so that the combination of the agents and the communication network satisfies the requirements on the overall system.

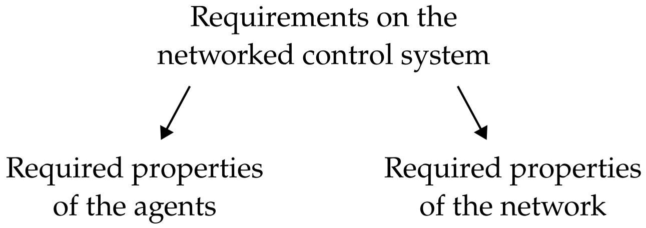

Fig. 1.2: Illustration of the controller design process

1.3 Fields of application

1.3.1 Autonomous vehicles

A common example concerning the control of networked systems are connected and automated vehicles (CAV). Autonomous driving is a highly discussed topic and is expected to improve traffic efficiency and safety [139]. Modern vehicles come with several driver assistance systems that are able to maintain a safety distance to the predecessor or to perform autonomous lane changes.

Vehicle platooning

Vehicle platooning describes the problem of arranging a set of vehicles in a line so that the inter-vehicle distances match a predefined (possibly velocity dependent) spacing. Adaptive cruise control (ACC) is a driver assistance system that should solve this task using sensor data only. Figure 1.3 shows the basic structure of an ACC platoon with vehicles that are equipped with radar sensors to measure the distance and a controller to adapt the velocity of each vehicle with the aim that a desired distance is achieved and all vehicles travel with a common velocity asymptotically. To benefit from reduced fuel consumption due to exploitation of slipstream, the distance has to be small which leads to the fact that a driver in an automated vehicle is not able to react on a disturbance in time to prevent an imminent collision. Thus, the cruise controller has to guarantee collision avoidance for an arbitrary length of the platoon.

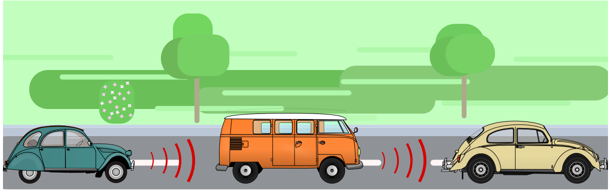

Fig. 1.3: Platoon with adaptive cruise control

An extension to ACC is cooperative adaptive cruise control (CACC) which uses the communication network in addition to the distance sensors. This approach allows for considering additional information from vehicles further downstream. Thus, the controlled vehicles can react faster on changes of the reference velocity due to traffic conditions or disturbances.

Vehicle merging

Vehicle merging can be interpreted as an extension of platooning which concerns the coordination of multiple platoons so that two or more platoons merge into a single one. Such techniques are required to pass highway on-ramps or lane reductions due to construction sites automatically. Commercially available approaches to the control of autonomous vehicles are currently based on the intensive use of sensor technology and estimation methods to predict the behaviour of other vehicles. Thus, lane changing assistants have to wait for a gap to occur coincidentally since there is no cooperation between vehicles. By introduction of a communication network, this limitation can be resolved.

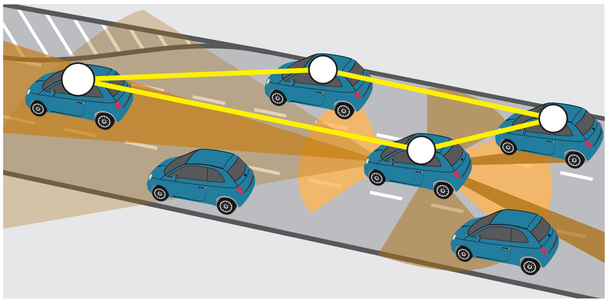

Fig. 1.4: Connected and automated vehicles passing through a lane reduction [86]

Figure 1.4 shows an example with multiple automated vehicles which are coupled in two ways. In addition to the measurement of the distance to the predecessor as required for platooning, the vehicles are equipped with sensors to detect the environment and other vehicles as illustrated by the cones. Furthermore, the local vehicle controllers can communicate with each other via wireless communication links (illustrated by the yellow edges) to exchange information and to perform cooperative manoeuvres if necessary.

With adaptive cruise controllers, the communication system is used only when a cooperation of multiple vehicles is mandatory to perform a specific manoeuvre, for example, when there is no gap on the target lane that is sufficiently large. The vehicles on the target lane then receive a request from the merging vehicle and generate a gap cooperatively for the merging vehicle to steer in.

1.3.2 Swarms of mobile systems

Motivated by nature, the collective behaviour of flocks of birds or ants serves as a model for swarms of mobile systems. The goal is to create patterns or formations that serve an overarching goal without the use of a centralised coordinator.

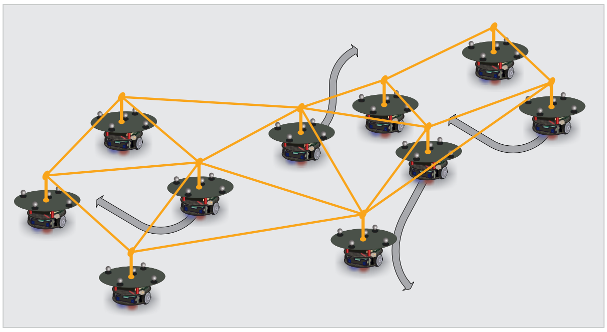

Fig. 1.5: A swarm of networked robots

Figure 1.5 shows a swarm of robots that is connected via communication links. The networked control of mobile agents has a wide area of applications to solve coverage, logistics or satellite control problems. The fundamental common task in these applications is to create desired formations by networked controllers.

In contrast to control problems concerning connected and automated vehicles, swarms of mobile agents are not bound to move on fixed roads but rather are able to move freely on a plane. This degree of freedom in the movement has to be considered by the networked controller to achieve the formations while guaranteeing collision avoidance.

1.4 Open questions

For the solution of the presented applications, two essential questions have to be examined.

- **Design of the local controllers:** What properties do the controlled vehicles have to have so that they can be combined in any way and in any number to achieve an overarching control aim?
- **Design of the communication structure:** When does a vehicle have to communicate with which other vehicles so that all vehicles can avoid collisions together?

These questions have been answered partially for vehicle platoons in [84, 88]. It has been shown that the vehicles in a platoon have to be *externally positive* in order to guarantee collision avoidance. However, the design of externally positive control loops is not solved in general [6]. The present thesis starts at that point and will address and solve the design of adaptive cruise controllers that render the controlled vehicle externally positive.

When considering vehicle platoons, the communication structure is often given as a path graph. In this thesis, the general question of an appropriate communication structure will hence be addressed for abstract swarms of vehicles that are not bound to prescribed paths as in Fig. 1.5. The vehicles are free to move on the surface and relatively to one another. Thus, the a communications structure that has been purposeful in the beginning can become unsuitable as the vehicles move along their trajectories. In order to find an appropriate communication structure that adapts to the current geometrical configuration of the vehicles, a proximity network based on the Delaunay triangulation will be studied as the communication structure of a networked system with mobile agents.

The following selection of references is based on the aspects that have been addressed in this section.

1.5 Literature overview

1.5.1 Networked control systems

In the field of networked systems, the most fundamental problem is the *consensus problem* with simple integrator dynamics where all agents should agree on a common steady state. It is shown that the graph-theoretic interpretation of the communication structure must include a spanning tree to achieve this goal [107]. The stability of networked systems is often studied in combination with communication effects as packet losses or delays [147].

An extension to higher-dimensional systems with the goal of bringing all control variables onto a common trajectory defines the *synchronization problem* which is often studied for subsystems with identical dynamics [125]. For the synchronization of heterogeneous systems, the internal-reference principle was presented in [81] and [82] which shows that the subsystems to be synchronised must necessarily have common dynamics.

1.5.2 Control of networked vehicles

Vehicle platooning has been shown to potentially increase efficiency and safety in daily traffic [139]. There are several recent papers that study energy-optimal adaptive cruise control from an ecological point of view [59, 60, 74].

One of the earliest works on vehicle platooning considered centralised optimal control techniques [73]. However, due to the decentralised character of traffic problems, decentralised methods to satisfy safety requirements are necessary for practical applications. To this end, methods of the control of networked systems have been applied to the control of connected and automated vehicles [40, 121, 122].

An important and fundamental requirement considering the control of vehicles is collision avoidance. Thus, in [132], the concept of *string stability* has been introduced and it has been shown that the string stability condition cannot be satisfied using distributed controllers in combination with a constant distance policy [127]. Thus, the idea of a time-dependent inter-vehicle distance was analysed, for example, in [68] which was applied in various theoretical and practical studies [64, 99, 103, 114, 146]. All these publications use a specific vehicle model with integrating dynamics and apply weak string stability to ensure safety requirements by shaping the closed-loop transfer function so that its magnitude is bounded. However, as has been proven in [84], the stricter property of external positivity has to be imposed on the controlled vehicles to guarantee collision avoidance. External positivity implies strong string stability because an externally positive vehicle is also $\mathcal{L}_\infty$ string stable [37].

The experimental results presented in [46, 98] have shown that commercially available vehicles are not externally positive at that time. Thus, the design of adaptive cruise controllers that render the controlled vehicles externally positive requires further investigations.

1.5.3 Externally positive systems

There are several books and articles on positive linear systems, e. g. [39, 61, 79, 118] which are based on the theory of positive matrices that goes back to Perron and Frobenius [24].

Despite being a less restrictive property, the literature on *externally positive systems* is far less comprehensive since external positivity is hard to characterise by the parameters of a system. To this end, some effort has been conducted based on the observation that externally positive systems have a nonovershooting step response. An interesting observation was published in [67] giving an upper bound for the number of extrema of the step response in terms of the location of the zeros of a system. This work motivated several authors to present necessary or sufficient conditions that avoid overshooting step responses [57, 78, 117, 133]. However, some of the conditions in these publications are either too complicated to be used to design control loops or are too restrictive with respect to the location of the eigenvalues [57]. Simpler sufficient conditions for external positivity were published in [58, 117] which are based purely on the configuration of the poles and zeros of a system.

There are several fields of application of positive systems in economics, population analysis, probabilistic models and compartmental systems [49]. In the monograph [39], a whole chapter is dedicated to the application of externally positive systems. Some recent publications showed that (external) positivity of the subsystems is a helpful property in the control of networked systems which simplifies the analysis of the stability and the consensus under constraints such as uncertain or probabilistic communication structures [25, 27, 86, 138]. Furthermore, external positivity is a property of controlled vehicles that can guarantee collision avoidance under well defined conditions [37, 84].

Even though there is a wide area of applications, there are only a few publications on the design of externally positive systems since the design problem is rather difficult. This is due to the fact that external positivity is characterised by a nonnegative impulse response which can be hardly determined analytically. Thus, the general design problem is unsolved [6]. In [134], a compensator is presented that renders a system externally positive but the contribution does not provide a constructive way for the feedback design problem. For multiple-input multiple-output systems, there are two publications that try to shape the eigenvectors of the controlled system so that each output is represented by one mode of the system which renders it externally positive. In [42], a linear matrix inequality was deduced that can be solved with optimal control techniques and a similar approach in [126] uses the modal synthesis to shape the eigenvectors. However, these approaches suffer from the limitation that the design method is only applicable to systems with unit relative degree which restricts the field of application. Some constructive sufficient conditions for external positivity based on a factorisation of the transfer function were published in [56, 58, 117].

1.5.4 Design of communication structures

The design of communication structures for networked control systems is a problem that has not been solved yet in general [86]. However, some effort for special control applications has been conducted, for example, in [88] where the design of a communication structure for platoons with CACC is elaborated. The results are based on a delay measure introduced in [83] that allows for transforming the design problem of a network structure into an algebraic problem. Furthermore, there are approaches that find an optimal communication topology with respect to the overall system performance by solving mixed-integer semidefinite programs [44, 48]. However, these methods are centralised in the design of the network topology and the network is fixed once it is determined.

The present thesis proposes the Delaunay triangulation as the network structure which should be maintained decentrally by the agents along their trajectories. The Delaunay triangulation is a well studied structure with a rich literature in the fields of computational geometry, geodesy, computer graphics, data clustering or spatial discretisation for finite element methods. It is the dual of the Voronoi diagram [52, 116] and is named after its inventor who addressed this topic in the original paper [35] where he defined his triangulation by its specific property that the unique circle through any three points connected as a triangle does not contain any other point in its interior. The Delaunay triangulation can be constructed for a given Voronoi diagram or directly for a given set of points [72]. A popular method for generating the Delaunay property is the Lawson flip which allows for adjusting two adjacent triangles if the empty-circumcircle property is violated [70, 71]. Conditions to test this property are given in [45, 52, 116].

The Delaunay triangulation belongs to the class of *proximity graphs* [95]. The structure of such graphs is determined by the relative position of its vertices. There are publications on networked systems that seek consensus using proximity graphs, e. g. [21] and [31] where the authors used r-disk graphs and integrators to model the agents. Some other applications use the Delaunay triangulation to evaluate and improve the placement of the vertices, e. g. ad-hoc networks or coverage control problems which seek an optimal distribution of the vertices in a given area [32, 75, 96]. The Delaunay triangulation has also been applied for formation control of multiple unmanned aircrafts [28] or path planning for autonomous vehicles [20]. In the field of computational geometry, the maintenance of the Voronoi diagram or the Delaunay triangulation of moving points was discussed in [19, 41, 130, 148] with centralised methods. The application of the Delaunay triangulation as a network structure for multi-agent systems requires the maintenance to be distributed so that it can be performed cooperatively by the agents which will be investigated in this thesis.

1.5.5 Swarms of mobile agents

A famous publication by Vicsek et al. [141] presented a study of particles that change their direction of movement by applying nearest neighbour rules. They showed that all particles agree on a direction under specific parameter values. This observation shows that it is possible to coordinate groups of subsystems without a superior coordinator which is desirable in the distributed control of mobile agents.

Distance-based formations as an example have been studied in various publications with linear (double) integrator models or nonholonomic models [36, 144]. In the related field of *flocking*, the agents should form a group in which the agents also align their velocity vectors so that the group travels together in a consensual direction [54, 106].

In order to solve such formation control problems, gradient-based control schemes with artificial potential fields (sometimes referred to as edge-tension functions, Lyapunov-like functions or barrier functions) are predestined due to the target-oriented influence on the agents [51, 97]. Since the gradient-method is well suited for the control of vehicle swarms, it will be applied in this thesis.

Collision avoidance as a fundamental requirement is often achieved with unbounded control inputs which result from the application of diverging potential functions [38, 93, 94, 119, 135–137]. While, theoretically, these schemes guarantee provable collision avoidance, they are not reliable in practice since control inputs are always bounded.

The application of artificial potential fields for formation control and collision avoidance on linear integrator models was discussed in several publications, e. g. in [38, 93, 106, 108, 143]. It is possible to extend these results to nonholonomic kinematics by application of a feedback linearisation [51, 119]. The direct application of the potential field method on nonholonomic kinematics was, for example, discussed in [50, 77, 94, 112, 113].

Control application with swarms of mobile agents usually apply a switching communication structure due to the individual movement of the agents. Related literature has shown that for the standard consensus protocol with integrator dynamics, consensus is achieved if the switching communication structure is always strongly connected and balanced which is true for any connected undirected graph [109]. The requirement has been relaxed so that the switching communication does not always have to possess a spanning tree but rather frequently enough [120]. These results also have been discussed and extended in the context of flocking in [54, 135–137] with nearest-neighbour rules.

By introduction of a switching communication graph, the resulting control system is a hybrid system with continuous dynamical states of the agents and discrete states of the communication graph. There are different tools to evaluate the stability of hybrid systems, e. g. multiple Lyapunov functions [29, 30, 34, 76] or a single common Lyapunov function that is applied to all discrete states [91] or nonsmooth analysis [129, 137]. However, these methods are difficult to apply as there are no systematic ways to construct the corresponding Lyapunov functions. Furthermore, it has been shown that there are switched linear systems that are stable but there does not exist a common Lyapunov function since the existence of a common quadratic Lyapunov function is only sufficient for stability. Some effort to prove stability of flocking algorithms with switching networks or nearest neighbour interactions with linear integrator dynamics can be found in [135–137].

1.6 Contributions of this thesis

The present thesis will address both the design of local controllers that implement desired properties of the agents and the choice of an appropriate communication network. The results (lemmas, theorems and algorithms) of the author are presented in boxes. Results that are taken from the literature are presented without a box marked with the corresponding reference. The main contributions of this thesis are described in the following.

1.6.1 Design of externally positive feedback loops

It has been shown that external positivity is a valuable property for various applications. However, the design of controllers that render a given plant externally positive is an unsolved problem [6]. This observation is due to the fact that external positivity of a linear system is characterised by its impulse response which is hardly determinable in dependence of the controller parameters analytically [100].

This thesis will summarise known conditions for external positivity in order to discuss approaches to solve the problem of designing controllers that result in an externally positive feedback loop. Theorem 3.4 reveals that plants with multiple eigenvalues equal to zero are not positively stabilisable.

Thus, in order to design adaptive cruise controllers, a state-feedback approach is proposed to solve the design problem for a general class of vehicle models. Sufficient conditions on the closed-loop eigenvalues that achieve externally positive closed-loop dynamics are given by Theorem 4.4. A design procedure for adaptive cruise controllers, which guarantee collision avoidance by rendering the controlled vehicles externally positive, will be developed and summarised by Algorithm 4.2. The effectiveness of the proposed algorithm is verified in an experimental environment in Section 7.3.

1.6.2 Maintenance of a proximity communication network

Swarms of mobile agents as an abstraction of real traffic problems require the communication structure to adjust to the current geometrical configuration of the agents. The Delaunay triangulation is proposed as a good choice of a communication structure for mobile agents due to the following reason. It has always a spanning tree and it has the important property that the pairwise closest agents are always connected which is an important property concerning the collision avoidance [95].

This thesis proposes to use the Delaunay triangulation as the communication network for the control of mobile agents. To this end, it will been shown that it is possible to maintain the Delaunay triangulation by the decentralised Algorithm 5.4 that is executed by all agents. The proposed algorithm is based on an efficient local characterisation of the Delaunay triangulation given by Theorem 5.1.

1.6.3 Collision avoidance in swarms of mobile systems

As the third theoretical contribution, this thesis studies the combination of a switching Delaunay triangulation network structure with local controllers that should achieve a distance-based formation. In contrast to the related literature that uses fixed communication structures and unbounded control inputs, this thesis will present a way to guarantee collision avoidance under consideration of control input limitations by the application of the Morse potential function [101] which has been first applied to control problems in [128].

With the parametrisation of the Morse potential function by Theorem 6.1, collision avoidance will be shown by Theorem 6.3 if the Delaunay triangulation communication structure is maintained during the transition of the agents. The result is summarised in Algorithm 6.1 which gives explicit instruction to apply the proposed controller in combination with the Delaunay triangulation network.

1.6.4 Experimental evaluation of the theoretical results

The theoretical results of this thesis are evaluated with laboratory experiments using differentially driven robots. The experimental results complement the theoretical results with further insights into practical aspects of the control applications.

In order to test the proposed adaptive cruise controller, a path tracking controller is developed that steers the robot to a prescribed path as shown by Theorem 7.1. The combination of simultaneous lateral and longitudinal control is studied with vehicle platoons that should follow a circular path or merge into a single platoon in case of a lane reduction. The experiments verify that collision avoidance in the platoon is achieved with external positive dynamics in contrast to platoons with $\mathcal{L}_2$ string stable vehicles that end up in a collision.

The remaining experiments consider swarms of robots that should form a distance-based formation and, as an extension, the transition problem is studied that describes the task of reaching an individual destination point while avoiding collisions with other robots. The experiments reveal that the proposed algorithms that maintain the Delaunay triangulation are real-time capable. Furthermore, the characterisation of the Morse potential achieves collision avoidance in the laboratory environment which verifies the theoretical results.

1.7 Structure of this thesis

The main results of this thesis are presented in Chapters 3 – 7. In the following, a brief overview of the content of each chapter is given.

Chapter 2 gives an overview of fundamental control-theoretic methods that will be applied in this thesis.

Chapter 3 introduces externally positive systems and summarises known properties and characteristics of such systems. Furthermore, it will be discussed for which classes of systems there exists a controller that renders the feedback loop externally positive or not.

Chapter 4 addresses the platooning problem. Several requirements are given that represent the desired behaviour of the overall platoon. Based on these requirements, design objectives are developed that should be achieved by local controllers. The design problem is then solved for a general class of vehicle models. In particular, a method to design externally positive vehicles using a state feedback is proposed.

Chapter 5 concerns the maintenance of a proximity communication network based on the Delaunay triangulation. First, the Delaunay triangulation is introduced and local conditions are developed to characterise the current network from the perspective of an agent. Based on these conditions, a method to test and adjust the current network is developed so that it represents a Delaunay triangulation after each iteration.

Chapter 6 solves the task of forming distance-based formations using gradient-based control techniques. It will be shown how to parametrise Morse potential functions that guarantee collision avoidance while respecting control input limitations. The results are extended to the case that a switching Delaunay triangulation network is applied and it will be shown under which conditions the collision avoidance is preserved.

Chapter 7 presents measurements that verify the theoretical results of the previous chapters using mobile robots. To this end, a lateral path controller is developed that complements the longitudinal platooning controller. Furthermore, the proposed swarming methods are evaluated with robots that are free to move on the driving surface.

Chapter 8 summarises the presented results.

Preliminaries and fundamentals 2

This chapter summarises fundamental models, methods and concepts of control theory that are relevant for the subsequent chapters.

2.1 Notation

Matrices and vectors are typeset in boldface (e. g. $\boldsymbol{A}$ or $\boldsymbol{x}$). The norm of a vector is denoted by $\left\|\boldsymbol{x}\right\|$ which represents the Euclidean norm if not specified otherwise. The determinant of a matrix is denoted by $\left|\boldsymbol{A}\right|$. For a scalar argument, $\left|a\right|$ denotes the absolute value. Furthermore, inequalities with vectors as $\boldsymbol{x} \geq 0$ are understood element wise.

Functions of time t are denoted by lower-case letters (e. g. $g(t)$) and the corresponding function in the Laplace domain by the same upper-case letters (e. g. $G(s)$). The Laplace transform is symbolized by $\circ\!\!-\!\!\bullet$ as in the example $g(t) \circ\!\!-\!\!\bullet G(s)$. The limit of a function is abbreviated as

$$\lim_{t \to \infty} g(t) = g(\infty).$$

The functions $\sigma(t)$ and $\delta(t)$ denote the Heaviside (unit step) function and the Dirac (impulse) distribution, respectively.

An overset exclamation mark as in the examples

$$h(t) \overset{!}{\geq} 0, \quad t \geq 0, \qquad\qquad h(0) \overset{!}{=} 0$$

indicates a condition that *should* be satisfied by an appropriate choice of parameters.

The gradient of a multivariable scalar function $f(\boldsymbol{x}) = f(x_1, x_2, \ldots, x_n)$ is denoted by $\nabla f(\boldsymbol{x})$ and is defined as the column vector of the partial derivatives

$$\nabla f(\boldsymbol{x}) := \begin{pmatrix} \frac{\partial f(\boldsymbol{x})}{\partial x_1} & \frac{\partial f(\boldsymbol{x})}{\partial x_2} & \ldots & \frac{\partial f(\boldsymbol{x})}{\partial x_n} \end{pmatrix}^{\mathrm{T}}.$$

The gradient with respect to a part of the argument, e. g. $\tilde{\boldsymbol{x}}^{\mathrm{T}} = \begin{pmatrix} x_1 & x_2 \end{pmatrix}$ is denoted by

$$\nabla_{\tilde{\boldsymbol{x}}} f(\boldsymbol{x}) := \begin{pmatrix} \frac{\partial f(\boldsymbol{x})}{\partial x_1} & \frac{\partial f(\boldsymbol{x})}{\partial x_2} \end{pmatrix}^{\mathrm{T}}.$$

Sets are denoted by calligraphic letters as $\mathcal{E}$. The empty set is denoted by $\emptyset$ and the cardinality of a set is given by $\left|\mathcal{E}\right|$ which represents the number of elements of a set.

2.2 Description of linear time-invariant systems

2.2.1 Description in the time domain

This thesis considers linear time-invariant systems denoted by Σ. The input and the output of Σ are denoted by the scalar signals $u(t)$ and $y(t)$, respectively. The dynamical behaviour of Σ is described by the n-th order ordinary differential equation

$$\begin{aligned}\Sigma: \frac{\mathrm{d}^n y(t)}{\mathrm{d}t^n} &+ a_{n-1}\frac{\mathrm{d}^{n-1} y(t)}{\mathrm{d}t^{n-1}} + \ldots + a_1 \frac{\mathrm{d}y(t)}{\mathrm{d}t} + a_0\, y(t) \\ &= b_q \frac{\mathrm{d}^q u(t)}{\mathrm{d}t^q} + b_{q-1}\frac{\mathrm{d}^{q-1} u(t)}{\mathrm{d}t^{q-1}} + b_1 \frac{\mathrm{d}u(t)}{\mathrm{d}t} + \ldots + b_0\, u(t)\end{aligned} \tag{2.1}$$

with constant coefficients $a_i, (i = 0, 1, \ldots, n{-}1)$ and $b_i, (i = 0, 1, \ldots, q)$. A system Σ that satisfies $n \geq q$ is called *proper* or *strictly proper* if $n > q$ holds. A system that is not proper has no technical realisation. In a diagram, a linear system is represented by a block as illustrated in Fig. 2.1.

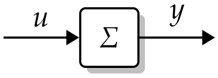

Fig. 2.1: Graphic representation of a linear system

For $n > q$, the differential equation (2.1) can be cast in the state-space representation

$$\Sigma: \begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{b}\,u(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ y(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t) \end{cases} \tag{2.2}$$

with $\boldsymbol{x}(t)$ denoting the *state* of Σ [89]. The state-space model (2.2) is abbreviated as $\Sigma = (\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}^{\mathrm{T}})$. The parameters $\boldsymbol{A}$, $\boldsymbol{b}$ and $\boldsymbol{c}^{\mathrm{T}}$ can be constructed for a given set of coefficients $a_i, (i = 0, 1, \ldots, n{-}1)$ and $b_i, (i = 0, 1, \ldots, q)$ and characterise the dynamical input-output behaviour completely. Note that the parameters $\boldsymbol{A}$, $\boldsymbol{b}$ and $\boldsymbol{c}^{\mathrm{T}}$ are not unique, i. e. there is an infinite number of sets of parameters that describe Σ equivalently.

The input-output behaviour of Σ is alternatively characterised by the impulse response

$$g(t) = \boldsymbol{c}^{\mathrm{T}}\mathrm{e}^{\boldsymbol{A}t}\boldsymbol{b},$$

which is the output of Σ if the input is the Dirac impulse: $u(t) = \delta(t)$.

2.2.2 Description in the frequency domain

The state-space model (2.2) is a time-domain representation of Σ. By introduction of the Laplace transforms $Y(s) \bullet\!\!-\!\!\circ\, y(t)$ and $U(s) \bullet\!\!-\!\!\circ\, u(t)$, an equivalent frequency domain representation of the input-output behaviour is given by

$$\Sigma: \; Y(s) = G(s)\, U(s)$$

with the *transfer function* $G(s)$ that is determined by the elements of (2.2) as

$$G(s) = \boldsymbol{c}^{\mathrm{T}}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}.$$

The transfer function $G(s)$ is the Laplace transform of the impulse response $g(t)$ of Σ. For a given set of parameters $a_i, (i = 0, 1, \ldots, n-1)$ and $b_i, (i = 0, 1, \ldots, q)$ (cf. (2.1)), the transfer function can be constructed as

$$G(s) = \frac{b_q s^q + b_{q-1} s^{q-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}.$$

The frequencies s that render the numerator or the denominator equal to zero are called *transmission zeros* s_{0i} or *poles* s_i of Σ, respectively. If they are known, the transfer function is equivalently given as the factorisation

$$G(s) = \kappa \, \frac{\prod_{i=1}^{q}(s - s_{0i})}{\prod_{i=1}^{n}(s - s_i)}$$

with some scalar constant κ. All poles s_i of $G(s)$ are furthermore also eigenvalues λ_i of $\boldsymbol{A}$.

2.3 Fundamentals of control theory

2.3.1 Controllability

Consider a strictly proper plant $\Sigma = (\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}^{\mathrm{T}})$. The question of whether Σ can be influenced so that a specified control aim is achieved (e. g. bring the output $y(t)$ to a desired value) is characterised by the following property.

Definition 2.1 (Controllability [90])**.** A system Σ is said to be *controllable* if there is an input $u(t), (t \in [0, t_{\mathrm{e}}])$ that moves the state $\boldsymbol{x}(t)$ from any initial state $\boldsymbol{x}_0$ to any given final state $\boldsymbol{x}(t_{\mathrm{e}})$ in final time t_{e}.

In order to test a given model for the controllability property, there are different necessary and sufficient conditions [90]. The criterion proposed by Kalman in [62] uses the *controllability matrix* which is defined as

$$\boldsymbol{S} := \begin{pmatrix} \boldsymbol{b} & \boldsymbol{A}\boldsymbol{b} & \cdots & \boldsymbol{A}^{n-1}\boldsymbol{b} \end{pmatrix}. \tag{2.3}$$